

**Theoretical Physics V – Addendum to Exercise Sheet 14**  
***Phase-invariance of the Dirac field and the Noether current***

(Dated: July 23, 2024)

*This additional exercise will be presented by one of the exercise leaders at the class on Thursday, 25.07.2024. We encourage you to try this exercise on your own beforehand if you have time.*

**ADDITIONAL EXERCISE**

Consider the Lagrangian density of a Dirac field  $\psi$ :

$$\mathcal{L} = \bar{\psi}(i\hbar\gamma^\mu\partial_\mu - mc)\psi. \tag{1}$$

- a) Show that the system is invariant under the phase transformation  $\psi \rightarrow \psi' = e^{i\alpha}\psi$ .
- b) Determine the Noether current  $j^\mu$  associated with the phase-invariance of the Lagrangian density.
- c) Show that the Noether current satisfies  $\partial_\mu j^\mu = 0$ .