# Theoretical physics V
## Sheet 11

**SoSe 2024** **Due for the 04.07.2024**

## Exercise 29 *Light-matter interaction: Rayleigh and Thomson scattering*

During the lectures, you broached the topic of light-matter interaction with a focus on light-emission by matter (spontaneous emission) and the renormalization of the mass of electrons by quantum fluctuations (Lamb shift). Here, we propose to go one step further by exploring the quantum description of the scattering of light by matter. More precisely, we will focus on the Rayleigh and Thomson regimes of light-scattering.

a) In attachment, you may find an extract of the textbook "Advanced Quantum Mechanics" by J. J. Sakurai (Chapter 2 - Section 5, pages 47-53). Using the informations contained in this material, prepare a short presentation to be realized at the blackboard. You are expected to present the derivation of the Kramers-Heinsenberg formula before applying it to the Rayleigh regime.

*(6 points)*

b) Present also the case of the Thomson scattering.

*(2 points)*

**Notes:**

- In the reference, Sakurai uses a formulation of electrodynamics in terms of the Lorentz units. In this system of units, the Maxwell equations take the form

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \rho \ \ ; \ \ \boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t}$$
$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0 \ \ ; \ \ \boldsymbol{\nabla} \times \boldsymbol{B} = \frac{1}{c}\boldsymbol{j} + \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t}.$$

- In this framework, the minimal coupling Hamiltonian reads

$$\hat{H} = \frac{1}{2m}\left(\boldsymbol{p} - \frac{q}{c}\boldsymbol{A}\right)^2 + q\phi,$$

where $\phi$ stands for the electric potential, while $\boldsymbol{A}$ is the vector potential such that $\boldsymbol{E} = -\boldsymbol{\nabla}\phi - \frac{1}{c}\partial_t \boldsymbol{A}$ and $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$.

- Therefore, for an atomic electron, the interaction with light is taken into account via a perturbative term of the form

$$\hat{H}_{\text{int}} = -\frac{q}{2mc}\left(\boldsymbol{p} \cdot \boldsymbol{A} + \boldsymbol{A} \cdot \boldsymbol{p}\right) + \frac{q^2}{2mc^2}\boldsymbol{A} \cdot \boldsymbol{A}.$$

- We shall note the equivalence between the notation used by Sakurai and the one used in the lecture for
$$|1_\lambda\rangle \equiv |\boldsymbol{k}, \epsilon^{(a)}\rangle$$
where $\lambda \equiv (\boldsymbol{k}, \epsilon^{(a)})$.

- In Sakurai's book, equation (2.124) gives the following relation
$$\langle B|\boldsymbol{p}|A\rangle = \frac{im}{\hbar}\langle B|[H_0, \boldsymbol{x}]|A\rangle = -\frac{im(E_B - E_A)}{\hbar}\langle B|\boldsymbol{x}|A\rangle$$

- We remind the definition of the diffraction function
$$\delta^{(T)}(E_f - E_i) = \frac{1}{2\pi\hbar}\int_{-T/2}^{T/2} \mathrm{d}\tau e^{i(E_f - E_i)\tau/\hbar}.$$

  Therefore, for a system perturbed by an interaction $\hat{V}$, up to second order in perturbation theory the elements of the scattering matrix take the form
$$S_{fi}(t) = \delta_{fi} - 2\pi i \left[V_{fi} + \lim_{\eta\to 0^+}\sum_k \frac{V_{fk}V_{ki}}{E_i - E_k + i\eta}\right]\delta^{(t)}(E_f - E_i),$$
  with $V_{fi} = \langle f|\hat{V}|i\rangle$.