Theoretical physics V
Sheet 8

SoSe 2024 **Due for the 13.06.2024**

Exercise 22 *Limit of the perturbative treatement*

In Exercise 18, we compared the prediction of a perturbative expansion at first order in a Dyson series with the actual exact descrition of the time-evolution of a system. In particular, we showed that is only valid at short times and fails to capture some aspects of the dynamics, such as oscillations. In the following, we shall discuss the validity of the perturbative treatement over long time-evolutions.

Attached to this sheet, you may find an extract of the textbook " *Quantum Mechanics - volume two*" by Claude Cohen-Tannoudji, Bernard Diu and Franck Laloë (chapter XIII - pages 1291-1297). Using the informations contained within this material, prepare a short presentation (to be realized at the blackboard). In this preparation, you shall detail the calculation and the reasoning leading to the estimation of the validity of the perturbative approach.

(6 points)

Conventions used in the material

- For an initial state $|i\rangle$ at energy $E_i = \hbar\omega_i$ and a final state $|f\rangle$ at energy $E_f = \hbar\omega_f$, the matrix element of the operator $\hat{W}$ reads
$$W_{fi} = \langle f|\hat{W}|i\rangle.$$
- The transition frequency between state $|i\rangle$ and $|f\rangle$ is noted
$$\omega_{fi} = \omega_f - \omega_i.$$
- Let us a Hamiltonian $\hat{H}_0$ such that $\hat{H}_0|\varphi_n\rangle = E_n|\varphi_n\rangle$, then add a perturbative terms such that the Hamiltonian becomes
$$\hat{H} = \hat{H}_0 + \lambda\hat{W}(t),$$
where λ quantifies the importance of this perturbation.

  Within this framework, the state of the system after a time t can be written in the following manner
$$|\psi(t)\rangle = \sum_n c_n(t)|\varphi_n\rangle,$$
with $c_n(t) = b_n(t)e^{-iE_nt/\hbar}$. The time-evolved form of the coefficients $b_n(t)$ can be expanded perturbatively in terms of the parameter λ:
$$b_n(t) = b_n^{(0)}(t) + \lambda b_n^{(1)}(t) + \mathcal{O}(\lambda^2).$$
Up to first order, the coefficients are given by the expressions
$$b_n^{(0)}(t) = c(0)$$
$$b_n^{(1)}(t) = \frac{1}{i\hbar}\int_0^t e^{i\omega_{ni}\tau}W_{ni}(\tau)\mathrm{d}\tau.$$