Chapter 2

Antenna Fundamentals

2.1 Introduction

2.1.1 Electromagnetic Spectrum, Radio Waves

Table2.1.1 shows that the electromagnetic spectrum is divided into several regions. Their boundaries are not precisely defined; rather, they are determined by changes in main physical characteristics, which are gradual, and also by the state of technology. In particular, there is a spectral overlap of radio waves and infrared light.

The term *radio frequency* (RF) refers to the frequency range up to 3 THz, corresponding to free-space wavelengths down to 100 μm. That range is divided into a number of bands which are listed in Table 2.1.2.

Example 2.1.1 (ELF waves: Schumann resonances). The surface of the earth, mainly salt water, and the ionosphere constitute two conducting surfaces. The air region in between forms a spherical cavity resonator which supports a discrete set of so called *Schumann resonances*. Schumann resonances are excited by lightnings. The first resonances are 7.8 Hz, 14.3 Hz, and 20.8 Hz. The mean circumference of the earth is ≈ 40000 km. The height of the conducting layer of

Table 2.1.1: The electromagnetic spectrum.

Name Unit	Freespace Wavelength m		Frequency Hz		Photon Energy eV
Radio waves	$< \infty$	$100 \cdot 10^{-6}$	> 0	$3 \cdot 10^{12}$	$< 12.4 \cdot 10^{-3}$
Infrared light	10^{-3}	$800 \cdot 10^{-9}$	$300 \cdot 10^{9}$	$375 \cdot 10^{15}$	< 1.55
Visible light	$800 \cdot 10^{-9}$	$400 \cdot 10^{-9}$	$375 \cdot 10^{15}$	$750 \cdot 10^{15}$	< 3.1
Ultraviolet light	$400 \cdot 10^{-9}$	$10 \cdot 10^{-9}$	$750 \cdot 10^{15}$	$30 \cdot 10^{15}$	< 124
X rays	$10 \cdot 10^{-9}$	$10 \cdot 10^{-12}$	$30 \cdot 10^{15}$	$30 \cdot 10^{18}$	$< 124 \cdot 10^{3}$
γ rays	$10 \cdot 10^{-15}$		$30 \cdot 10^{21}$		

Table 2.1.2: Radio frequency bands.

Frequency		Code	Name	Main Usage
3 – 30	Hz	ELF	Extremely Low Frequency	
30 – 300	Hz	SLF	Super-Low Frequency	Power grids
300 – 3000	Hz	ULF	Ultra-Low Frequency	Aircraft power
3 – 30	kHz	VLF	Very Low Frequency	Submarine communication
30 – 300	kHz	LF	Low Frequency	Beacons
300 – 3000	kHz	MF	Medium Frequency	AM broadcast
3 – 30	MHz	HF	High Frequency	AM broadcast
30 – 300	MHz	VHF	Very High Frequency	FM broadcast, TV
300 – 3000	MHz	UHF	Ultra High Frequency	TV, cellular, LAN
3 – 30	GHz	SHF	Super High Frequency	Radar, LAN, satellites, data
30 – 300	GHz	EHF	Extremly High Frequency	Radar, data

the ionosphere may be assumed at 300 km (maximum of electron density during day and night.)

2.1.2 Radio Communication Link

Fig. 2.1.1 shows a simple wireless communication link comprising a transmitter, a wireless channel, and a receiver. The transmitter consists of an RF generator, a feed line, and an antenna that converts the guided wave coming from the feed to a radiating waveform in open space. In the receiver, the antenna serves the opposite purpose: to convert a free-space wave to a guided wave on a feed line which leads to an RF amplifier. Note that

- *Antennas* are devices for radiating or receiving radio waves; but:

- *Antennae* are sensitive organs of insects.

At GHz frequencies, a physical limitation for applications in wireless communications is given by the atmospheric attenuation of electromagnetic waves due to water vapor and absorption by gases [9]; see Fig. 2.1.2.

2.1.3 Network Representation

In typical applications, the transmitter and receiver antennas are so far apart that the fraction of radiated power accepted by the receiver is very small. While such links would be very inefficient for power transfer, they may still be very useful for information (data) exchange. It is characteristic of such configurations that the presence or absence of the receiver does not affect the overall characteristics of the transmitter, such as antenna input impedance or far-field patterns.

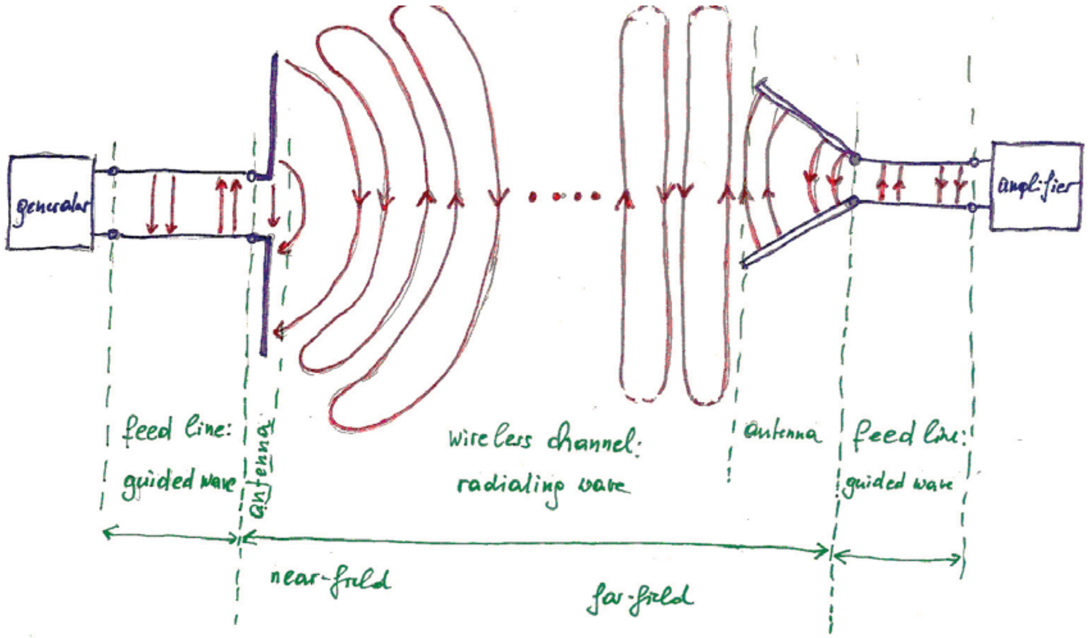


Figure 2.1.1: A simple wireless communication link.

An equivalent circuit for the transmitter is shown in Fig. 2.1.3: The generator is modeled by a voltage source of source impedance Z_S, the feed line is characterized by its length, propogation coefficient, and wave impedance, and the antenna by some load impedance. Its resistive part is caused by the power radiated into space and the losses inherent to the antenna. Its reactive component corresponds to the energy stored in the near-field around the antenna.

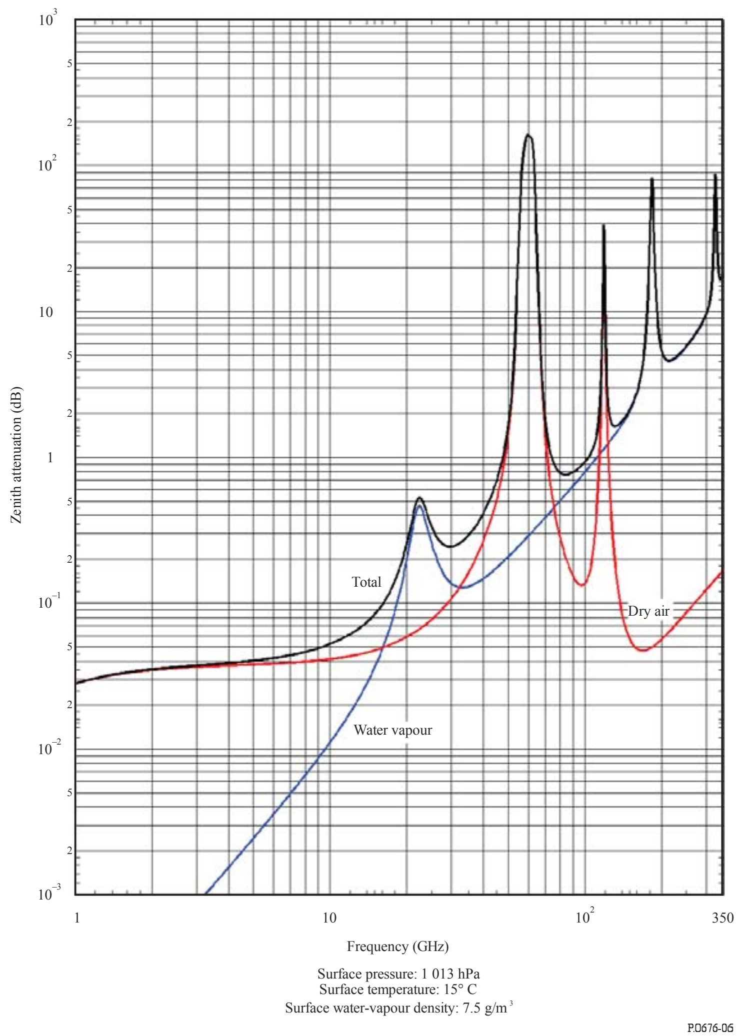


Figure 2.1.2: Atmospheric attenuation versus frequency. Reproduced from [9].

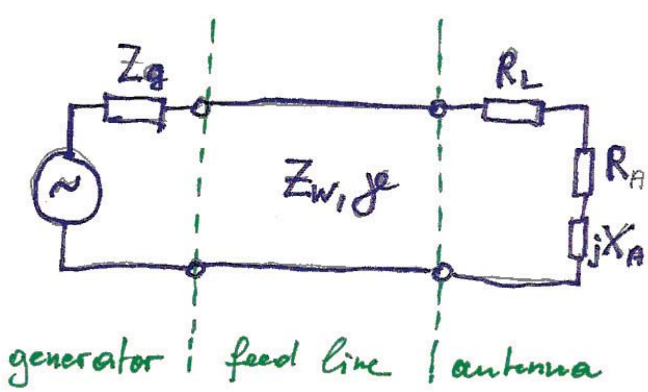


Figure 2.1.3: Equivalent circuit model of transmitter.

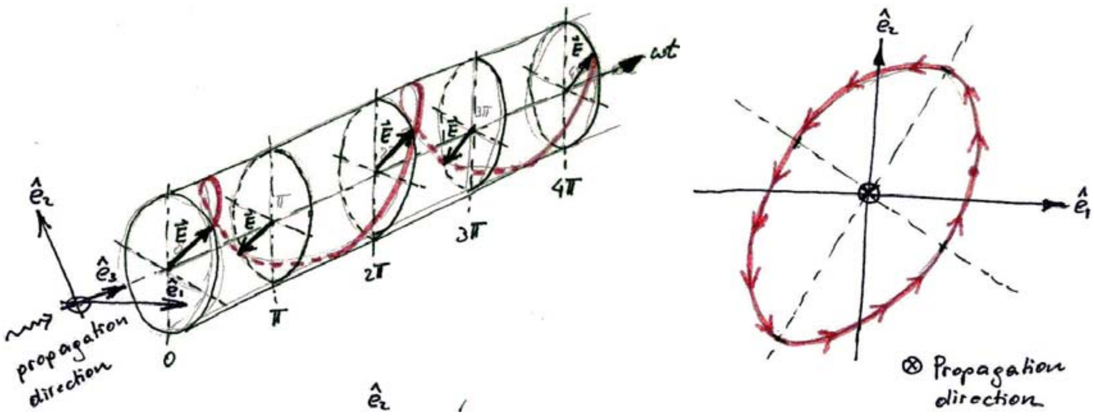


Figure 2.2.1: Elliptical polarization of an EM wave.

2.2 Polarizaton of Locally Plane Wave

Definition 7 (Polarization of a locally plane wave)**.** The property that describes the time-varying direction and relative magnitude of the $\vec{E}$ vector; specifically, the figure traced as a function of time by the extremity of the vector at a fixed location in space, and the sense in which it is traced, as observed along the direction of propagation. [10, p. 64]

Linear polarization means that the electric field vector as a function of time moves along a straight line. For a given direction of propagation, there exist two orthogonal, linearly polarized waves. Any wave propagating in this direction can be written as a superposition of two linearly polarized waves of specific magnitude and phase. If the phase difference between the linearly polarized waves is non-zero, the resulting wave is of *elliptical* polarization: In general, the projection of $\vec{E}$ as a function of time on the transverse plane is a tilted ellipse; see Fig. 2.2.1.

Definition 8 (Axial ratio (AR))**.** The ratio of the major axis to the minor axis is called the *axial ratio* (*D: Achsenverhältnis*).

A special case is *circular polarization*, in which case the AR is one. In case of elliptical or circular polarization, *right-hand polarization* means that $\vec{E}$ rotates in the clockwise sense, and *left-hand polarization* designates counter-clockwise rotation.

Any plane wave can be decomposed into one left-handed and one right-handed circularly polarized wave propagating in the same direction, each of specific amplitude and phase.

Exercise 2.2.1. Given one left-hand and one right-hand circularly polarized wave, of same frequency, and propagating in the same direction. Prove that the waves are orthogonal in the sense that they are energetically decoupled.

Exercise 2.2.2. Prove that any circularly polarized wave can be represented by the sum of a left-hand and a right-hand circularly polarized wave.

Exercise 2.2.3. Prove that any linearly polarized wave can be represented by the sum of a left-hand and a right-hand circularly polarized wave.

Exercise 2.2.4. Let the time-harmonic electric field at a given point be described by the phasor $\vec{E} = |E_1|e^{j\varphi_1}\hat{\mathbf{e}}_1 + |E_2|e^{j\varphi_2}\hat{\mathbf{e}}_2 + |E_3|e^{j\varphi_3}\hat{\mathbf{e}}_3$. What type of surface is traced by the instantaneous field over one period? What is the maximum of the instantaneous field over time, and at what phase instance does it occur?

2.3 Antenna Parameters

2.3.1 Bandwidth

Definition 9 (Bandwidth (BW))**.** The range of frequencies within which the performance of the antenna conforms to a specified standard with respect to some characteristic:

$$BW = f_{\max} - f_{\min}. \tag{2.3.1}$$

Often the relative bandwidth is used:

$$\text{rel. BW} = \frac{f_{\max} - f_{\min}}{\bar{f}} \qquad \text{with } \bar{f} = \frac{f_{\max} + f_{\min}}{2}. \tag{2.3.2}$$

Typical characteristics include pattern, impedance, polarization, etc..

2.3.2 Polarization of Antenna

Definition 10 (Polarization of antenna)**.** The polarization of an antenna in a given direction is that of the plane wave it *radiates* at large distances in that direction.

If the direction of the polarization is not stated, the direction of maximum gain is implied.

For non-reciprocal antennas which are not intended to transmit, it makes sense to define the receiving polarization as *the polarization of a plane wave, incident from a given direction, which results in maximum available power at the antenna port*. If the receiving polarization is meant, this must be clearly specified.

Definition 11 (Co-polarization (*D: Kopolarisation*))**.** That polarization which the antenna is intended to radiate.

Definition 12 (Cross-polarization (*D: Kreuzpolarisation*))**.** In a specified plane containing the reference polarization ellipse, the polarization orthogonal to a specified reference polarization.

The reference polarization is usually the co-polarization.

2.3.3 Antenna Input Resistance

Definition 13 (Input resistance R (*D: Eingangswiderstand*))**.** The *input resistance* is the resistance measured at the terminals of the antenna.

Let i denote the current at the location of the terminals. Then the input resistance is obtained from the radiated power P_{rad} by

$$P_{rad} = \frac{1}{2} R_{rad}\, |i|^2. \tag{2.3.3}$$

Example 2.3.1 (Input resistance of Hertzian dipole). According to (1.4.37), the total radiated power is given by $P_{rad} = \frac{|i|^2}{2} \frac{2\pi}{3}\eta_0 \left(\frac{|\Delta l|}{\lambda}\right)^2$. Since the current along the Hertzian dipole is constant,

$$R = R_{rad} = \frac{2\pi}{3}\eta_0 \left(\frac{|\Delta l|}{\lambda}\right)^2. \tag{2.3.4}$$

2.3.4 Radiation Patterns

The purpose of antennas is not only to excite radiating waveforms but also to direct the radiated power to selected directions: Most antennas are *directional radiators*. The *radiation pattern* or *antenna pattern* is defined as a mathematical function or a graphical representation of the radiation properties as a function of space coordinates. Since radiation patterns are defined for the far-field zone, they are most commonly expressed in terms of the directional spherical coordinates of the observation point, the *look angles* ϑ and φ.

Radiation patterns may be defined for a variety of physical quantities. The most common choices are power flux density $T_r(\vartheta, \varphi)$ and electric field strength $\vec{E}(\vartheta, \varphi)$. Whereas T_r is a real-valued scalar, $\vec{E}(\vartheta, \varphi)$ is a complex-valued vector. Thus, to fully describe $\vec{E}$ fields, a total of four patterns is required: one for the magnitude and phase of each of the two transverse components. The importance of $\vec{E}$ patterns lies in the fact that many antennas are polarization-dependent, i.e., the strength of the signal received depends on the relative orientation of the antenna with respect to the direction of $\vec{E}$.

Most often, magnitude patterns are normalized with respect to the respective maximum assumed over all look angles, e.g.:

$$\text{normalized power pattern:} \qquad p_{norm}(\vartheta, \varphi) = \frac{T_r(\vartheta, \varphi)}{T_r(\vartheta, \varphi)_{\max}}, \tag{2.3.5}$$

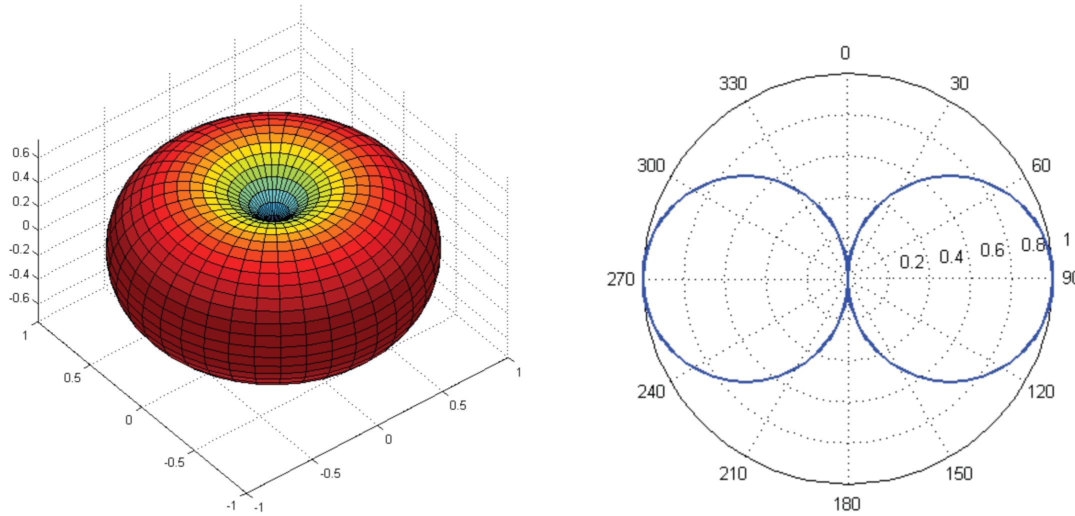
$$\text{normalized field pattern:} \qquad E_{\vartheta,norm}(\vartheta, \varphi) = \frac{|E_\vartheta(\vartheta, \varphi)|}{|E_\vartheta(\vartheta, \varphi)|_{\max}}. \tag{2.3.6}$$

Normalized patterns are dimensionless ratios which are often represented by logarithmic measures, in decibel (dB). For consistency, the calculation must reflect the origin as a power or field ratio, respectively:

$$\text{logarithmic measure of } p_{norm} = 10 \log_{10} p_{norm}(\vartheta, \varphi) \qquad \text{in dB}, \tag{2.3.7}$$

$$\text{logarithmic measure of } E_{\vartheta,norm} = 20 \log_{10} E_{\vartheta,norm}(\vartheta, \varphi) \qquad \text{in dB}. \tag{2.3.8}$$

Definition 14 (Isotropic radiator)**.** A hypothetical lossless antenna having equal radiation in all directions.



(a) Three-dimensional view. (b) Elevation plane.

Figure 2.3.1: Hertzian dipole $i\Delta l\hat{\mathbf{e}}_z$: linearly scaled normalized FF patterns of E_ϑ.

Since the radiation intensity of an isotropic radiator U_{iso} does not depend on the look angles, we have

$$P_{rad} = \oint_\Omega U_{iso}\,\mathrm{d}\Omega = U_{iso}\oint_\Omega \mathrm{d}\Omega = 4\pi U_{iso}, \tag{2.3.9a}$$

$$U_{iso} = \frac{P_{rad}}{4\pi}. \tag{2.3.9b}$$

Definition 15 (Omnidirectional radiator)**.** An antenna having an essentially nondirectional pattern in a given plane.

Principal Planes

Definition 16 ($\vec{E}$ plane, $\vec{H}$ plane, principal planes)**.** The $\vec{E}$ plane ($\vec{H}$ plane) of a linearly polarized antenna is the plane containing the electric (magnetic) field vector and the maximum radiation. The $\vec{E}$ plane and the $\vec{H}$ plane are referred to as the principal planes.

It is common use to choose the coordinate system such that at least one of its principal planes coincides with the principal planes of the radiation pattern. For antennas providing linear polarization, it often suffices to characterize them by two two-dimensional patterns in their principal planes.

Example 2.3.2 (FF Pattern of Hertzian Dipole)*.* The Hertzian dipole radiates linearly polarized waves. The $\vec{H}$ plane of the antenna is given by its equatorial plane. By symmetry, the $\vec{E}$ plane is not unique; it is represented by any plane containing the dipole. The radiation pattern of the dipole is omnidirectional in the $\vec{H}$ plane but directional in the $\vec{E}$ plane; see Fig. 2.3.1.

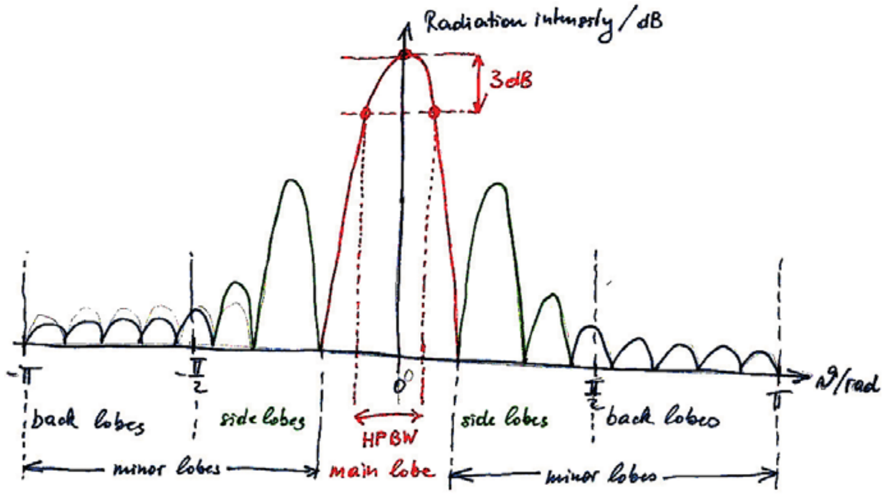


Figure 2.3.2: Classification of radiation lobes.

Antenna Lobes

Definition 17 (Radiation lobe (*D: Strahlungskeule, Antennenkeule*))**.** A portion of the radiation pattern bounded by regions of relatively weak radiation intensity.

The radiation patterns of directional antennas feature one or more lobes. The *main lobe*, *major lobe* or *main beam* is the lobe containing the direction of maximum radiation. All others are called *minor lobes*; The standard defines a *sidelobe* as a lobe in any direction other than the intended lobe. In practical use, sidelobes are the larger, unwanted minor lobes adjacent to the main lobe, in forward direction. The *sidelobe level* is the ratio of the maximum power density of the sidelobe to that of the main lobe, usually expressed in db.

A *back lobe* is defined as defined as a radiation lobe the axis of which makes an angle of approximately 180 degrees with respect to the beam axis of an antenna lobe in the half-space opposed to the direction of peach directivity. Loosely speaking, a back lobe is any lobe radiating into the opposite half-space of the main lobe.

In a pattern cut containing the direction of the maximum of a lobe, the *half-power beamwidth* (HPBW) is given by the angle between the two directions in which the radiation intensity is one half the maximum value. If the major lobe has a half-power contour that is essentially elliptical, the *principal HPBWs* denote the HPBWs in the two pattern cuts that respectively contain the major and minor axes of the ellipse.

The *first-null beamwidth* (FNBW) is the angle between the two directions in which the radiation intensity has a null. In practise the nulls may be filled up due to phase errors. The FNBW may then be approximated by the angle between the two directions of minimum radiation.

Fig. 2.3.2 illustrates the definitions above.

2.3.5 Directivity and Beam Solid Angle

Definition 18 (Directivity D (*D: Richtfaktor, Direktivität*))**.** The directivity $D(\vartheta, \varphi)$ is the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions, i.e., the ratio to the radiation intensity of an isotropic radiator of same total radiated power:

$$D(\vartheta, \varphi) = \left.\frac{U(\vartheta, \varphi)}{U_{iso}}\right|_{P_{rad}=\text{const}} = 4\pi \frac{U(\vartheta, \varphi)}{P_{rad}}. \tag{2.3.10}$$

If the direction is not specified, the direction of maximum radiation intensity is implied.

Since the directivity is a dimensionless ratio that may vary greatly, it is common to give a logarithmic measure, in decibel over isotropic (dBi).

$$\text{logarithmic measure of } D = 10 \log_{10} D(\vartheta, \varphi) \qquad \text{in dBi.} \tag{2.3.11}$$

Definition 19 (Beam solid angle Ω_A, beam area (*D: äquivalenter Raumwinkel*))**.** The solid angle through which all the radiated power would stream if the power per unit solid angle were constant throughout this solid angle and at the maximum value of the radiation intensity $U_{\max}$:

$$\Omega_A := \frac{P_{rad}}{U_{\max}}. \tag{2.3.12}$$

By (2.3.12) and (2.3.10), the beam area is related to the maximum directivity by

$$\Omega_A = \frac{4\pi U_{iso}}{U_{\max}} = \frac{4\pi}{D_{\max}}. \tag{2.3.13}$$

Example 2.3.3 (Directivity and beam area of Hertzian dipole). The radiation intensity is given by (1.4.36) and the radiated power by (1.4.37). Eq. (2.3.10) yields

$$D(\vartheta, \varphi) = \frac{4\pi U}{P_{rad}} = 4\pi \frac{\frac{|\underline{i}|^2}{2} \frac{\eta_0}{4} \left(\frac{|\Delta \mathbf{l}|}{\lambda}\right)^2 \sin^2\vartheta}{\frac{|\underline{i}|^2}{2} \frac{2\pi}{3} \eta_0 \left(\frac{|\Delta \mathbf{l}|}{\lambda}\right)^2}, \quad D(\vartheta) = \frac{3}{2}\sin^2\vartheta, \tag{2.3.14}$$

$$D_{\max} = \frac{3}{2} \mathrel{\hat{=}} 1.76 \text{ dBi}, \tag{2.3.15}$$

$$\Omega_A = \frac{4\pi}{D_{\max}} = \frac{8\pi}{3} \text{ ster.} \tag{2.3.16}$$

2.3.6 Gain and Equivalent Isotropically Radiated Power

Definition 20 (Gain, absolute gain $(G(\vartheta,\varphi))$ (*D: absoluter Gewinn*))**.** The ratio of the radiation intensity in a given direction to the radiation intensity that would be produced if the *power accepted by the antenna* were isotropically radiated:

$$G(\vartheta,\varphi) := \frac{U(\vartheta,\varphi)}{P_{in}/(4\pi)}. \tag{2.3.17}$$

If the direction is not specified, the direction of maximum radiation intensity is implied.

Definition 21 (Relative gain $(g(\vartheta,\varphi))$ (*D: relativer Gewinn*))**.** The ratio of the gain of an antenna in a given direction (ϑ,φ) to the gain of a reference antenna in an agreed reference direction $(\vartheta_{ref},\varphi_{ref})$:

$$g(\vartheta,\varphi) = \frac{G(\vartheta,\varphi)_{\text{antenna under test}}}{G(\vartheta_{ref},\varphi_{ref})_{\text{reference antenna}}}. \tag{2.3.18}$$

Since gain is related to accepted rather than available power, it does not include reflection loss due to impedance mismatch of the feed. It is independent of the system the antenna is connected to. In contrast to directivity, gain accounts for dissipative losses within the antenna structure.

Definition 22 (Radiation efficiency (e_{cd}))**.** The ratio of the total power radiated by an antenna to the net power P_{in} accepted by the antenna from the connected transmitter:

$$e_{cd} := \frac{P_{rad}}{P_{in}}. \tag{2.3.19}$$

The indices emphasize that **c**onduction and **d**ielectric losses are included. Combining (2.3.19), (2.3.17), and (2.3.10) yields

$$G(\vartheta,\varphi) = e_{cd}D(\vartheta,\varphi). \tag{2.3.20}$$

Definition 23 (Equivalent Isotropically Radiated Power (EIRP))**.** In a given direction, the gain $G(\vartheta,\varphi)$ of a transmitting antenna multiplied by the net power P_{in} accepted by the antenna from the connected transmitter:

$$\text{EIRP}(\vartheta,\varphi) := G(\vartheta,\varphi)P_{in}. \tag{2.3.21}$$

2.3.7 Mismatch Mechanisms

Reflection Loss

In the transmission case, the antenna is excited through the feed by an incident wave of complex amplitude a_1; there is no incident wave from the antenna, $a_2 = 0$.

Unless the feed is perfectly wave-matched to the antenna, a reflected wave will be excited in the feed. Its amplitude b_1 is related to a_1 by the reflection coefficient ρ_1,

$$b_1 = \rho_1 a_1. \tag{2.3.22}$$

By definition of a and b, the available power P_{avail} and the reflected power P_{refl} are given by

$$P_{avail} = |a_1|^2, \tag{2.3.23}$$
$$P_{refl} = |b_1|^2 = |\rho_1|^2 P_{avail}. \tag{2.3.24}$$

Assuming that the junction is lossless, the power accepted by the antenna P_{in} is obtained from conservation of power:

$$P_{avail} = P_{in} + P_{refl}, \tag{2.3.25}$$
$$P_{in} = (1 - |\rho_1|^2) P_{avail}. \tag{2.3.26}$$

Definition 24 (Impedance mismatch factor (e_p))**.** The fraction of the available power that is accepted by the antenna:

$$e_p := \frac{P_{in}}{P_{avail}} = 1 - |\rho_1|^2, \tag{2.3.27}$$

wherein ρ_1 is the input reflection factor.

Definition 25 (Realized gain ($G_r(\vartheta, \varphi)$))**.** The ratio of the radiation intensity in a given direction to the radiation intensity that would be produced if the *power available at the input port* of the antenna were isotropically radiated:

$$G_r(\vartheta, \varphi) := \frac{U(\vartheta, \varphi)}{P_{avail}/(4\pi)}. \tag{2.3.28}$$

In contrast to gain, realized gain also accounts for the impedance mismatch factor. Eqs. (2.3.28), (2.3.27), (2.3.20) imply

$$G_r(\vartheta, \varphi) = e_p G(\vartheta, \varphi) = e_p e_{cd} D(\vartheta, \varphi). \tag{2.3.29}$$

Polarization Loss

Definition 26 (Polarization efficiency, polarization loss factor (PLF))**.** The ratio of the power received by an antenna from a given plane wave of arbitrary polarization to the power that would be received by the same antenna from a plane wave of the same power flux density and direction of propagation, the state of polarization of which has been adjusted for a maximum received power.

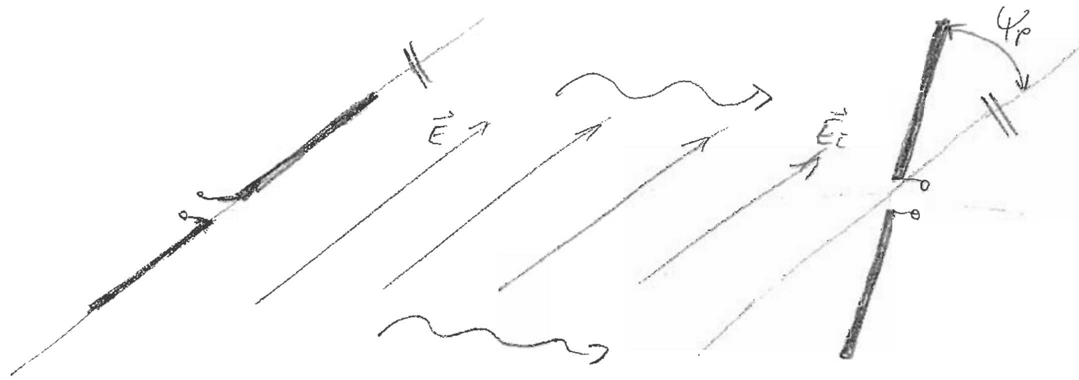


Figure 2.3.3: Polarization mismatch between a linearly polarized plane wave and a receiving dipole antenna.

Example 2.3.4. The polarization of the Hertzian dipole antenna is linear and in the direction of is axis; see Sec. 1.4.3. A linearly polarized plane wave is impinging upon a dipole pointing in the $\hat{\mathbf{e}}_r$ direction. The electric field vector at the location of the receiver is given by $\mathbf{E} = \hat{\mathbf{e}}_i E_i$; see Fig. 2.3.3. Determine the PLR.

The component of the incident electric field in the direction of the polarization of the antenna $E_\parallel$ is given by the projection

$$E_\parallel = \hat{\mathbf{e}}_d \cdot \hat{\mathbf{e}}_i E_i = \cos\psi_p E_i, \tag{2.3.30}$$

wherein ψ_p denotes the angle between the dipole axis and $\mathbf{E}$.

Since power is proportional to $|E|^2$, we have

$$\text{PLF} = \frac{|E_\parallel|^2}{|E_i|^2} = (\hat{\mathbf{e}}_d \cdot \hat{\mathbf{e}}_i)^2 = \cos^2\psi_p. \tag{2.3.31}$$

Exercise 2.3.1. A right-hand circularly polarized plane wave is received by a right-hand polarized antenna (per def.: transmission scenario!). Calculate the polarization loss factor.

2.4 Receiving Properties

2.4.1 Effective Antenna Length

Definition 27 (Effective length of a linearly polarized antenna)**.** For a linearly polarized antenna receiving a plane wave from a given direction, the ratio of the magnitude of the open-circuit voltage developed at the terminals of the antenna U_{oc} to the magnitude of the electric-field strength in the direction of the polarization of the antenna [11, p. 13].

It thus makes sense to define the vector equivalent length $\mathbf{l}_e$ [10, p. 79] by

$$\mathbf{l}_e : \qquad U_{oc} = \mathbf{E}_i \cdot \mathbf{l}_e. \tag{2.4.1}$$

The usefulness of the equivalent antenna length for dipoles and monopoles is evident. It may be used for all types of antennas, including loops and apertures, even though there is no immediate physical interpretation.

Exercise 2.4.1. Show that for a short dipole of length $l \ll \lambda/2$ the equivalent antenna length is equal to half its physical length.

Exercise 2.4.2. What is the equivalent length of a half-wave dipole?

2.4.2 Effective Antenna Area

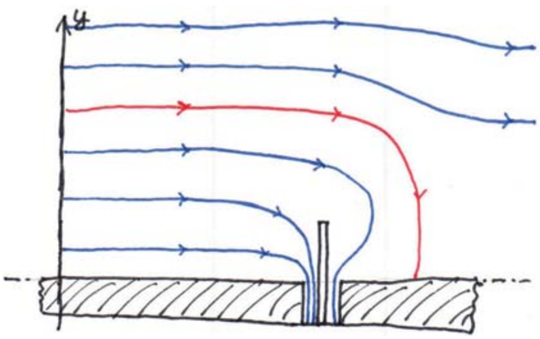
Definition 28 (Effective antenna area A_e (*D: Antennenwirkfläche*))**.** In a given direction, the ratio of the available power at the terminals of a receiving antenna to the power flux density of a plane wave incident on the antenna from that direction, the wave being polarization matched to the antenna. [11, p. 13]

$$A_e(\vartheta, \varphi) := \frac{P_{avail}(\vartheta, \varphi)}{T_r(\vartheta, \varphi)}. \tag{2.4.2}$$

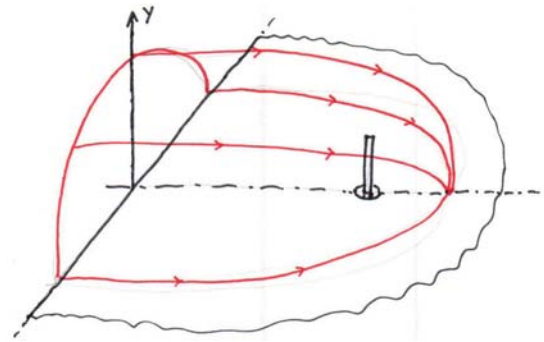
If the direction is not specified, the direction of maximum radiation intensity is implied. Synonymously, *equivalent aperture* may be used [10, p. 81].

For large aperture antennas, the equivalent area is equal to the physical area of the aperture. The usage of A_e is not restricted to aperture antennas, though; it may be computed via (2.4.2) for any kind of antenna.

In the general case, A_e may be interpreted as follows: Consider the entirety of all flux tubes of the power flux density (Poynting) vector $\mathbf{T}$ leading to the input terminal of the antenna. Their total flux is P_{avail}. At large distance from the antenna, the area A covered by the cross-sections of these flux tubes in a plane perpendicular to the direction of propagation is equal to A_e; see Fig. 2.4.1. The reason is as follows: At large distance from the antenna, the fields form a locally uniform plane wave. Thus the $\mathbf{T}$ flux tubes are in the direction of propagation (even if there is reflection from the antenna), and the magnitude of T is constant; hence $TA = P$.



(a) Red line separates flux lines entering the antenna from those passing by.



(b) At large distance, the cross-section of the flux tubes entering the antenna is equal to A_e.

Figure 2.4.1: Interpretation of equivalent area by means of the Poynting vector.

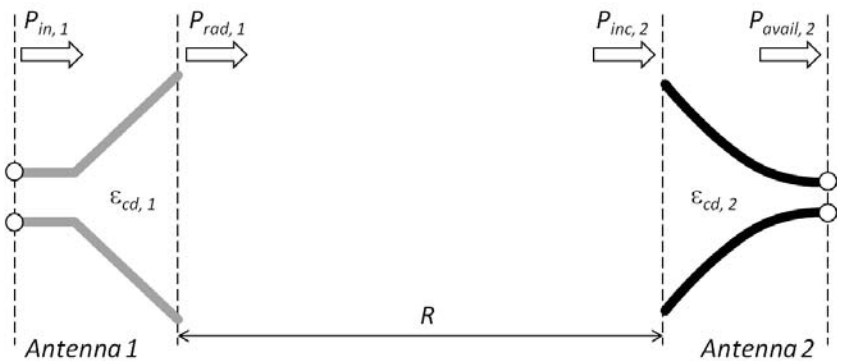


Figure 2.5.1: A simple wireless transmission path.

2.5 Wireless Transmission Path Properties

2.5.1 Equivalence of Transmitting and Receiving Characteristics

Consider the configuration of Fig. 2.5.1. When Antenna 1 is transmitting, and Antenna 2 is receiving, the power density T_{21} of the plain wave impinging on the receiver is related to the power accepted P_1 by the transmitting antenna by

$$T_{21} = \frac{G_1(\vartheta_1, \varphi_1)}{4\pi R^2} P_1^{in}. \tag{2.5.1}$$

Hence the power available at the output port of Antenna 2, neglecting polarization loss, is given by

$$P_{21}^{avail} = \frac{A_{e2}(\vartheta_2, \varphi_2) G_1(\vartheta_1, \varphi_1)}{4\pi R^2} P_1^{in}. \tag{2.5.2}$$

When Antenna 2 is taken as the transmitter and Antenna 1 as the receiver, while the locations of the antennas remain unaltered, we have

$$P_{12}^{avail} = \frac{A_{e1}(\vartheta_1, \varphi_1) G_2(\vartheta_2, \varphi_2)}{4\pi R^2} P_2^{in}. \tag{2.5.3}$$

Provided that both the antennas and the wireless channel are reciprocal,

$$\frac{P_{21}^{avail}}{P_1^{in}} = \frac{P_{12}^{avail}}{P_2^{in}}, \tag{2.5.4}$$

eqs. (2.5.2) and (2.5.3) imply

$$\frac{A_{e2}(\vartheta_2, \varphi_2)}{G_2(\vartheta_2, \varphi_2)} = \frac{A_{e1}(\vartheta_1, \varphi_1)}{G_1(\vartheta_1, \varphi_1)}. \tag{2.5.5}$$

Hence the ratio of equivalent area to gain is an invariant, whose value may be determined by reference to, e.g., a lossless Hertzian dipole, with $\varepsilon_{cd} = 1$. Its maximum directivity is known from (2.3.15), and its maximum equivalent area will be calculated in Sec. 3.0.3, in (3.0.21):

$$\frac{A_e}{G} = \frac{A_e}{\epsilon_{cd} D} = \frac{\frac{3\lambda^2}{8\pi}}{1 \cdot \frac{3}{2}} = \frac{\lambda^2}{4\pi}. \tag{2.5.6}$$

Hence, for any reciprocal antenna in any direction:

$$\frac{A_e(\vartheta, \varphi)}{G(\vartheta, \varphi)} \equiv \frac{\lambda^2}{4\pi}. \tag{2.5.7}$$

Since the ratio A_e/G does not depend on the look angles (ϑ, φ), the transmitting characteristics measured by $G(\vartheta, \varphi)$ and the receiving characteristics measured by $A_e(\vartheta, \varphi)$ of any reciprocal antenna are the same, but for a scaling constant.

Exercise 2.5.1. What is the equivalent antenna area of an isotropic radiator?

2.5.2 The Friis Transmission Equation

Eq. (2.5.2) relates the power available at the terminals of the receiving antenna to the power available at the input terminals of the transmitting antenna. Thanks to (2.5.5), it may be expressed in terms of G only:

$$P_{21}^{avail} = \frac{1}{\left(4\pi \frac{R}{\lambda}\right)^2} G_2 G_1 P_1^{in} \qquad \text{for polarization-matched antennas.} \tag{2.5.8a}$$

If the feeds are taken as the references and all losses are included, as shown in Fig. 2.5.2, one obtains

$$P_{feed2}^{avail} = \underbrace{\left(1 - |\rho_1|^2\right)\left(1 - |\rho_2|^2\right)}_{\text{reflection loss}} \underbrace{\left(\hat{\mathbf{e}}_d \cdot \hat{\mathbf{e}}_i\right)^2}_{\text{polarization loss}} \underbrace{\frac{1}{\left(4\pi \frac{R}{\lambda}\right)^2}}_{\text{free-space loss}} \underbrace{\epsilon_{cd,2}\epsilon_{cd,1}}_{\text{dissipative loss}} D_2 D_1 P_{feed1}^{avail}. \tag{2.5.8b}$$

Eqs. (2.5.8a) and (2.5.8a) are variants of the *Friis Transmission Equation*.

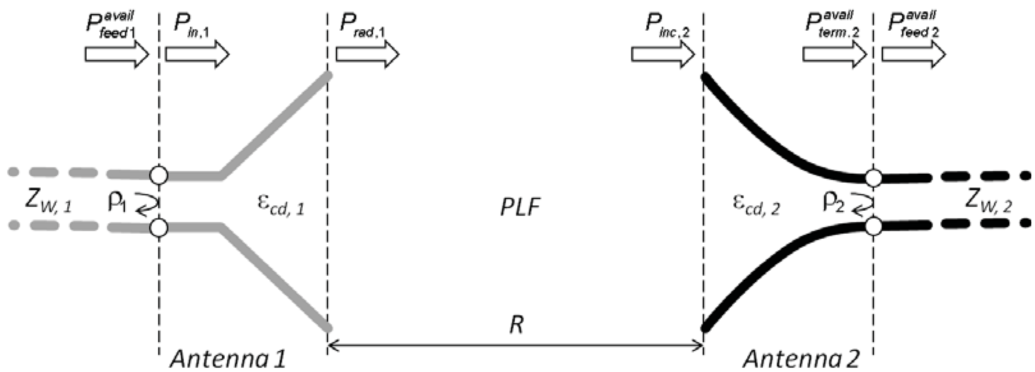
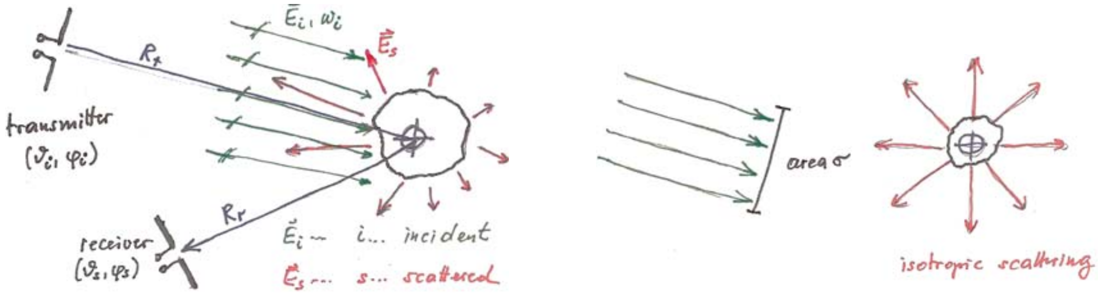


Figure 2.5.2: Wireless path including all losses.

Definition 29 (Free-space loss F (*D: Freiraumdämpfung*))**.** Free-space loss denotes the ratio of the power transmitted by an isotropic radiator to the power transmitted by an isotropic antenna, as a function of the electric distance $\frac{R}{\lambda}$ of the antennas. From (2.5.8a), its value is seen to be

$$F = \left(4\pi \frac{R}{\lambda}\right)^2 \quad \hat{=} \quad 20 \log_{10} \left(4\pi \frac{R}{\lambda}\right) \text{ dB}. \tag{2.5.9}$$

Free-space loss does not include dissipative processes in the atmosphere or reflections. Rather, it describes the decrease in power density of a homogeneous spherical wave as a function of the electric distance from the radiator.



(a) Actual radar path. (b) Equivalent model for definition of RCS.

Figure 2.6.1: Modeling the scattering properties of a radar target.

2.6 Radar Path Properties

Radar (*ra*dio *d*etection *a*nd *r*anging) denotes a broad class of methods for determining the distance (range), angular direction (azimuth and elevation angles), and speed of objects (targets) with the help of radio waves. The common principle is to illuminate the target by a radio wave and evaluate its response (echo), with regard to time delay, direction, magnitude, and Doppler shift. Thus a typical radar path involves a transmitting antenna, a scatterer (target), and a receiving antenna. For cost reasons, many radar systems employ the same antenna for transmitting and receiving. A comprehensive overview of radar techniques is given in [12].

2.6.1 Radar Cross-Section

Fig. 2.6.1a shows a scatterer about the center of the coordinate system, a transmitting antenna at $(R_1, \vartheta_i, \varphi_i)$, and a receiving antenna at $(R_2, \vartheta_s, \varphi_s)$. If the directions to the antennas coincide, $(\vartheta_i, \varphi_i) = (\vartheta_s, \varphi_s)$, the radar is *monostatic*; else *bistatic*. The incident wave impinging upon the target is partly absorbed and partly re-radiated, in various directions. That radiation from the target constitutes the *scattered field*. Thus the signal at the location of the receiver depends on the strength, frequency, and polarization $\hat{\mathbf{e}}_i$ of the incident wave, the distances R_i and R_s, the incidence and observation directions (ϑ_i, φ_i) and (ϑ_s, φ_s), and, of course, the characteristics of the target.

If the power density T_2 at the receiver site were due to a fictitious isotropic radiator at the target location, its total radiated power P_s would have to be

$$P_s = 4\pi R_2^2 T_2. \tag{2.6.1}$$

To afford that power, the target would need to intercept the power flow of the incident wave through the so-called *radar cross-section* σ,

$$P_s =: \sigma T_i, \qquad [\sigma] = \mathrm{m}^2, \tag{2.6.2}$$

Table 2.6.1: Typical RCS Values at Microwave Frequencies (after [12, p. 64])

Target	RCS in m^2	RCS in dBsm
Conventional winged missile	0.1	-10
Small single-engine aicraft	1	0
Helicopter	3	xx
Medium jet airliner	20	13
Jumbo jet	100	20
Small insect (fly)	1e-5	-50
Large bird	0.01	-20
Adult	1	0
Bicycle	2	3
Automobile	100	20
Small open boat	0.02	-17
Cabin cruiser	10	10
Large ship at zero grazing angle	$\geq$ 10 000	$\geq$ 40

where T_i denotes the power density of the incident wave at the target location. Fig. 2.6.1b illustrates the concept.

Definition 30 (Radar cross-section σ (RCS) (*D: Radarquerschnitt*))**.** The area intercepting that amount of power which, when scattered isotropically, produces at the receiver a power density which is equal to that scattered by the actual target.

Eqs. (2.6.1) and (2.6.2) imply

$$\sigma(\vartheta_s, \varphi_s, \omega; \vartheta_i, \varphi_i, \hat{\mathbf{e}}_i) = \lim_{R_2 \to \infty} 4\pi R_2^2 \frac{T_2(\vartheta_s, \varphi_s, \omega; R_2)}{T_i(\vartheta_i, \varphi_i, \hat{\mathbf{e}}_i)}. \qquad (2.6.3)$$

The monostatic RCS is commonly abbreviated by $\sigma_0(\vartheta, \varphi, \omega, \hat{\mathbf{e}}_i)$. Since RCS values may vary widely, they are often given in logarithmic form, in decibel over square meter (dBsm). Def. 30 follows [10, p. 88], [12, p. 6], [2, p. 116] but does not conform with the Standard [11, p. 30, p. 33], which includes the polarization of the scattered wave. A selection of typical RCS values at microwave frequencies is given in Table 2.6.1. Fig. 2.6.2 shows the monostatic RCS of a propeller-driven aircraft in the azimuth plane.

2.6.2 The Radar Range Equation

We are now ready to calculate the power available at the terminals of the receiving antenna P_2^{avail} from the input power of the transmitting antenna P_1^{in}. For brevity, let us consider the polarization-matched case:

$$P_2^{avail} = A_{e,2} T_2 = A_{e,2} \frac{1}{4\pi R_2^2} P_s = A_{e,2} \frac{\sigma}{4\pi R_2^2} T_i = A_{e,2} \frac{\sigma}{4\pi R_2^2} \frac{G_1}{4\pi R_1^2} P_1^{in}. \qquad (2.6.4)$$

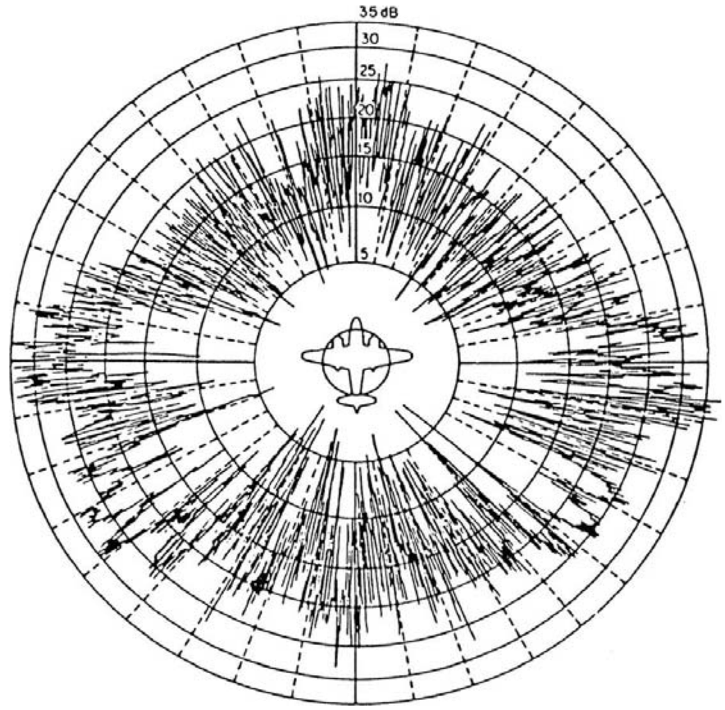


Figure 2.6.2: Monostatic RCS from B-26 two-engine propeller-driven aircraft at 3 GHz as a function azimuth angle. Reproduced from [12, p. 58].

Hence

$$\frac{P_2^{avail}}{P_1^{in}} = \frac{1}{(4\pi)^2}\frac{\sigma A_{e,2}}{R_2^2 R_1^2}G_1 = \frac{1}{(4\pi)^3}\frac{\sigma\lambda^2}{R_2^2 R_1^2}G_2 G_1 = \frac{1}{4\pi}\frac{A_{e,2}A_{e,1}}{R_2^2 R_1^2}\frac{\sigma}{\lambda^2}. \tag{2.6.5a}$$

In the monostatic case, using a single antenna, (2.6.5a) simplifies to

$$\frac{P_2^{avail}}{P_1^{in}} = \frac{1}{(4\pi)^2}\frac{\sigma A_e}{R^4}G = \frac{1}{(4\pi)^3}\frac{\sigma\lambda^2}{R^4}G^2 = \frac{1}{4\pi}\frac{A_e^2}{R^4}\frac{\sigma}{\lambda^2}. \tag{2.6.5b}$$

Eqs. (2.6.5a) and (2.6.5b) are variants of the *radar range equation*. It becomes apparent from (2.6.5b) that the received power decays with R^4, which is much faster than in the case of wireless transmission; see (2.5.8a).

Exercise 2.6.1. A perfectly conducting object is illuminated by a plane wave, with $\underline{\mathbf{E}}_i(\vec{r})$. Calculate the equivalent sources for the scattered field in free space.

Exercise 2.6.2. For targets that are electrically large and of complicated shape, the scattered fields in the FF region tend to be highly oscillatory functions of the observation direction. Give a qualitative explanation. (Hint: Use the results from Exercise 2.6.1.)