Weihnachtsworkshop 2024

16.12.-18.12.2024

Simon Barazer
Directed Ribbon Graphs and Acyclic Decomposition

Ribbon graphs are graphs embedded in surface with boundary. They were used in the proof of Witten conjecture, and more recently to compute Masur Veech volumes of principal strata in moduli spaces of quadratic differentials. I will focus on directed ribbon graphs and present a recursive structure that allows to compute volumes of combinatorial moduli spaces of directed metric ribbon graphs. We can call this structure the acyclic decomposition. It uses surgeries on ribbon graphs and it is similar to topological recursion. Afterwards we will see how the acyclic decomposition is related to Cut-and-Join equations for Hurwitz numbers of Grothendieck dessins d'enfants.

Laurent Bartholdi
Automata for the Thue-Morse Subshift

The Thue-Morse word 0110100110010110... appears in many guises in mathematics, but also signal processing, and even chess, thanks to its many fascinating properties (it does not contain any cube; any non-trivial shift differs in at least 1/3 of the positions; etc.). It is convenient to study it as a dynamical system, and even more to encode this dynamical system using automata. I will concretely present this encoding, and what can be deduced from it. This is one of the simplest examples on which a general theory applies, and I will explain in this special case the general results that hold for a large class of dynamical systems.

Thomas Dubbe
Zeros of the $\zeta-$-Function near 1 and The Large Sieve

We know, that there is no non-trivial zero $\rho = \beta + i\gamma$ of the Riemann $\zeta-$function with real part $\beta \geq 1 - \frac{r}{\log \gamma}$. The best known value for this (de la Valéé Poussin-type) zero-free region is due to Mossinghoff, Trudgian and Yang with $r \approx 0.1798$. Of course, one can ask how many zeros might appear between the heights T and $T + H$ if we increase r and thus moving the line $1 - \frac{r}{\log t}$ slightly to the left. In this talk we will explore how the Large Sieve is connected to this problem and how it can be used to derive explicit upper bounds for the number of such zeros.

Elia Fioravanti

The Pressure Metric on Quasi-Fuchsian Spaces

Thermodynamic formalism, as initially developed by Bowen and Ruelle, can be used to quantify the difference between two Anosov flows in terms of a "pressure semi-norm" on suitable spaces of Hölder functions. McMullen showed that the Weil-Petersson metric on Teichmüller space can be interpreted in these terms, after which "pressure metrics" were defined on broader representation varieties, for instance on quasi-Fuchsian spaces by Bridgeman, and on Hitchin components by Bridgeman, Canary, Labourie and Sambarino. While the infinitesimal structure at the Fuchsian locus is well-understood in these varieties, little is known on the global behaviour of pressure metrics. In joint work with Ursula Hamenstädt, Frieder Jäckel and Yongquan Zhang, we show that the pressure metric on quasi-Fuchsian spaces has finite diameter.

Manuel Kany

Arithmeticity of the Kontsevich–Zorich Monodromy of Origamis

An *origami* $\mathcal{O}$ or *square-tiled surface* is a compact Riemann surface obtained as a collection of finitely many unit squares in $\mathbb{R}^2$ with glued parallel edges by translations. The group $\mathrm{Aff}^+(\mathcal{O})$ consists of orientation preserving homeomorphisms φ on $\mathcal{O}$ which preserve the corners of the unit squares and away from them φ is locally given by an affine map. We want to study the action of $\mathrm{Aff}^+(\mathcal{O})$ on the singular homology $H_1(X, \mathbb{Q})$ by pull back, respectively the induced representation $\rho : \mathrm{Aff}^+(\mathcal{O}) \longrightarrow \mathrm{GL}(H_1(X, \mathbb{Q}))$. The homology $H_1(\mathcal{O}, \mathbb{Q})$ naturally decomposes into two $\mathrm{Aff}^+(\mathcal{O})$-invariant subspaces H_1^{st} and $H_1^{(0)}$. The subrepresentation $\rho|_{H_1^{st}}$ of ρ is easy to understand. Hence our goal is to understand the subrepresentation

$$\rho|_{H_1^{(0)}} : \mathrm{Aff}^+(\mathcal{O}) \longrightarrow \mathrm{GL}(H_1^{(0)}),$$

which is called the *Kontsevich–Zorich* monodromy. The representation ρ is closely related to the so called *algebraic monodromy* or *symplectic monodromy* of a certain holomorphic family of compact Riemann surfaces. Motivated by a conjecture of Griffiths and Schmid from 1974 we will focus on the question whether the image of the Kontsevich–Zorich monodromy $\rho|_{H_1^{(0)}}$ is an arithmetic group.

Benjamin Klopsch

Hausdorff Dimension in p-adic Analytic Pro-p Groups

In the theory of finitely generated pro-p groups G, certain canonical filtration series by open normal subgroups play a significant role. Each filtration series gives rise to a metric on G and a corresponding Hausdorff dimension function. The Hausdorff spectrum of G is a

subset of the real unit interval $[0,1]$ that displays, in a condensed form, information about all 'sizes' of closed subgroups in G, as detected by the filtration. Following an introduction to the subject, mainly based on previous joint work with A. Thillaisundaram and A. Zugadi-Reizabal (2019), I will discuss some recent discoveries about the Hausdorff spectra of p-adic analytic pro-p groups with respect to the lower p-series, which form part of joint work with I. de las Heras and A. Thillaisundaram (see arXiv:2402.06876). In particular, we show that the Hausdorff spectrum of a p-adic analytic pro-p group G, with respect to the lower p-series, is discrete and consists of at most 2^d rational numbers, where $d = \dim(G)$ denotes the analytic dimension of G. This result complements a corresponding theorem of Barnea and Shalev (1997) for the Hausdorff spectra of p-adic analytic pro-p groups with respect to the p-power filtration.
Interestingly, the new result displays rather more complexity and leads on to several open questions.

Bernhard Köck

Computing Modular Representations Given by Poly-Differentials on Drinfeld Curves

The action of $G = \mathrm{SL}(2, \mathbb{F}_q)$ on the Drinfeld curve was first investigated by Drinfeld in 1974. He noted that cuspidal representations (certain characteristic 0 representations) of G can be found in the first étale cohomology of this curve. Our work pertains to decomposing the characteristic p representations that come from this action on spaces of globally holomorphic poly-differentials. We first compute the decomposition for the subgroup of upper-triangular matrices of G, then determine the composition factors for the action of G and finally use intricate modular representation theory to find the decomposition into indecomposables for G. This is work of/with my PhD student Denver Marchment.

Bram Petri

Random hyperbolic surfaces with large systoles

The systole of a hyperbolic surface is the length of the shortest closed geodesic on that surface. How large the systole of a closed hyperbolic surface of a fixed genus can be is a classical question that is still mostly open. I will speak about joint work with Mingkun Liu on how random constructions can be used to build surfaces with large systoles.

Miguel Prado

The Isoresidual Curve on Genus Zero

I will talk about abelian differentials over the Riemann sphere with fixed orders and fixed residues at their poles. In particular, I will address the case of differentials with two zeroes which leads to an "isoresidual" curve. This curve can be compactified inside the multiscale space of differentials and expressed as a complete intersection of divisors. This allows us to compute its genus which can be described as a piecewise polynomial in terms of the orders of the zeroes and poles. This is joint work with Dawei Chen, Gendron and Tahar.

Anja Randecker

Ramsey properties for big mapping class groups

Big mapping class groups (that is, the symmetry groups of surfaces of infinite topological type) are interesting examples of Polish groups. So we could ask for big mapping class groups and their subgroups whether they are amenable, extremely amenable, or (closely related) whether they fulfil Ramsey properties.

This talk will show more strategies than results and is based on discussions with Jesús Hernández Hernández, Oscar Molina, Israel Morales, Jareb Navarro Castillo, Carlos Pérez Estrada, and Ulises Ariet Ramos-García.

Jan-Christoph Schlage-Puchta

A Monte-Carlo algorithm to estimate the index of the Veech group of an origami

An origami is a connected finite covering of an elliptic curve which has at most one branch point. Origami can be represented by a pair of permutations (π, σ) generating a transitive group. $\mathrm{Sl}_2(\mathbb{Z}) \cong \mathrm{Aut}(F_2)$ acts via Nielsen moves on the set of all pairs of permutation, the stabilizer of a permutation (π, σ) is the Veech group of this origami.

Schmithüsen gave an algorithm to compute the Veech group of an origami, which runs essentially linearly in the index of the Veech group in $\mathrm{Sl}_2(\mathbb{Z})$, that is, linearly in the size of the output. This is clearly best possible. Unfortunately the index of the Veech group of an Origami with n squares can be close to $n!$, so linear in the output is not sufficient for some questions.

Here we describe a Monte-Carlo algorithm that estimates the index of the Veech group of an origami in time close to the square root of the index. The algorithm uses ideas from biological statistics.

Johannes Schmitt

Euler Characteristics of Strata of k-Differentials

Inside the moduli space of smooth genus g curves C with n marked points p_1, .., p_n, the stratum ${H_g}^k(\mu)$ of k-differentials parameterizes the set of such curves carrying a k-differential with zero/pole order μ_i at the marking p_i. This definition makes sense for arbitrary integers k and $\mu_1, \ldots, \mu_n$ summing to the degree $k(2g-2)$ of the k-th cotangent bundle of C.

In the three regimes $k = 0$, $k = 1$ or $k > 1$, the spaces ${H_g}^k(\mu)$ have vastly different behaviours, parameterizing objects such as Hurwitz covers of the projective line, translation surfaces or $\frac{1}{k}$-translation surfaces. Nevertheless, I present a conjecture stating that the (orbifold) Euler characteristic of ${H_g}^k(\mu)$ is a polynomial of degree $2g$ in the entries μ_i of μ. Coming out of extensive computer experiments, this conjecture connects the three different regimes of k, and raises the possibility of finding a closed formula for this Euler characteristic.

In the talk, I give an introduction to the spaces of k-differentials, explain how to calculate their Euler characteristic in examples, and present the conjectural polynomial formulas in low genus.

Maximilian Wackenhuth

On sphere packing bounds conjectured by Cohn and Zhao

The current best upper bounds on sphere packing density are obtained by linear programming methods developed by Cohn and Elkies. Cohn and Zhao conjectured that these linear programming methods generalize to sphere packings in hyperbolic space. In this talk we will see how one can obtain linear programming bounds for sphere packing density in hyperbolic space. Our main technical tools are spherical harmonic analysis and the theory of density for packings in hyperbolic space developed by Bowen and Radin.

Ivan Yakovlev

Metric ribbon graphs

Metric ribbon graphs are graphs embedded into surfaces whose edges are equipped with some positive lengths. They are equivalent to meromorphic differentials with real periods. One naturally comes to the problem of their enumeration when studying random square-tiled surfaces, for example. I will show how bijections from the theory of combinatorial maps can help obtain explicit enumerative formulas for these objects. This combinatorial approach complements the approach via intersection theory, and it might be interesting to understand the connection between the two.